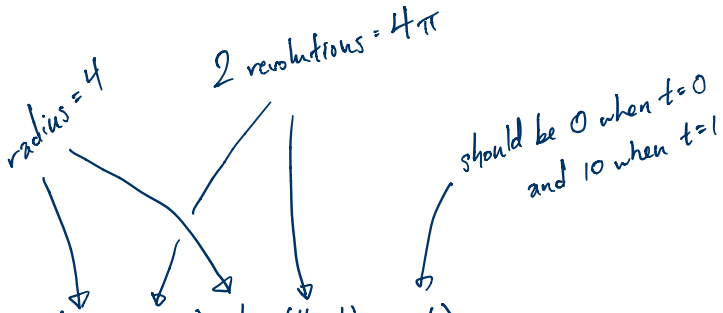
Name: SOLUTIONS Student ID:

MATH 53, QUIZ 4

Be sure to show neat, organized, complete work in the space provided.

1. A helix (spiral) centered around the z-axis starts at the point $(4, 0, 0)$ when $t = 0$, makes 2 full revolutions, and ends at the point $(4, 0, 10)$ when $t = 1$.

   (a) There are TWO such helixes. Give the parametrization of the one which winds *counterclockwise* when viewed from above. Make sure your parametrization is for $0 \le t \le 1$, as stated above.



Answer: $\vec{r}(t) = \langle 4\cos(4\pi t), 4\sin(4\pi t), 10t \rangle$

   (b) What is the arclength of the helix?

$$\int_0^1 |\vec{r}\,'(t)|\,dt = \int_0^1 \sqrt{(16\pi)^2 + 10^2}\,dt$$

Answer: $\sqrt{(16\pi)^2 + 100}$

2. Consider the function $f(x,y) = \dfrac{3x^3}{x^2+y^2}$ as (x,y) approaches $(0,0)$.

(a) By switching to polar coordinates, evaluate the limit if it exists, or explain why it does not exist. Make sure to fully justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{3x^3}{x^2+y^2} = \lim_{\substack{r\to 0^+ \\ \theta=\text{anything}}} \frac{3r^3\cos^3\theta}{r^2} = \lim_{\substack{r\to 0^+ \\ \theta \text{ anything}}} 3r\cos^3\theta = 0.$$

$-3r \le 3r\cos^3\theta \le 3r \quad (r>0)$

and $\lim_{\substack{r\to 0^+ \\ \theta \text{ anything}}} (-3r) = 0 = \lim_{\substack{r\to 0^+ \\ \theta \text{ anything}}} 3r$ so Squeeze Thm. says

(b) In view of your answer to the preceding part, is it possible to extend f to a continuous function g whose domain is the entirety of $\mathbb{R}^2$? If not, why not? If so, what would the value of $g(0,0)$ be?

Yes. Let $g(0,0) = 0$, i.e. g is defined by $g(x,y) = \begin{cases} \dfrac{3x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

Feedback: If you have any feedback you'd like to share, please write it here. If there are any specific topics you are confused about, feel free to write them here as well.